1. Find the Householder transformation matrix which maps the column vector $(1, 1, 1, 1, 1, 1)^T$ into a vector of the form $(1, 1, 1, *, 0, 0)^T$ and determine the value of the fourth element of the second vector marked by $*$. 

Given that any Householder transformation matrix is an orthogonal matrix the two vectors must have the same Euclidean norm. Therefore that fourth element in the second vector must be $\sqrt{3}$ or $-\sqrt{3}$. 

The Householder matrix is of the form $I - 2\frac{vv^T}{v^Tv}$ where $v$ is simply the difference between the two vectors, i.e., $v = (0, 0, 0, -\sqrt{3} - 1, -1, -1)^T$. 

Cross out what is not meant to be part of your answers.
2. (a) Give an example of a linear system of equations that has fewer equations than unknowns and which does not have any solution.

(b) Given an example of a linear system of equations that has fewer equations than unknowns, has a solution, and write down all its solutions.

(c) Are there any such linear systems that have a unique solution? Explain your answer.

(a) \( x_1 + x_2 + x_3 = 0, \ x_1 + x_2 + x_3 = 1 \), is clearly inconsistent and without any solution.

(b) If there is a solution, then any other solution will differ from it by an element in the null space of the matrix. There is always a null space given that the columns of any matrix with more columns than rows must be linearly dependent. All solutions can also be found by using an appropriate selection of the unknowns as free variables and solving a square linear system for the remaining variables. We create that square matrix by selecting a subset of the columns of the given matrix which are linearly independent.

(c) See the discussion above. We never have a unique solution.