1. Consider Newton’s method and the Illinois algorithm for finding roots of $f(x) = 0$ where $f(x)$ is a continuous function that changes sign at least once.

List at least two reasons why the Illinois algorithm is preferable to Newton’s.

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In each step of the Newton algorithm, the value of the first derivative has to be computed while the Illinois algorithm only requires function values.

The Illinois algorithm always converges provided we can find two values of $x$ with a sign change in between. There are cases when Newton’s method will fail to converge.

After each step of the Illinois algorithm, we have an upper and a lower bound for the root; something similar is not available for Newton’s method. Therefore, it is much easier to develop a reliable and simple stopping criteria for the Illinois algorithm.
2. A principal result for Newton’s method is that we have convergence provided that the initial guess is close enough to a root.

(a) It is also known that we have rapid convergence provided that a certain condition holds. What is that condition?

The first derivative of $f'(x)$ should differ from zero at the root.

(b) Construct a polynomial, which changes sign when we pass through a root and for which Newton’s method will converge slowly.

We need a polynomial with a first derivative which should vanish at the root and also change sign. We cannot work with a double root since we would not have a change of sign, but a triple root would do. The simplest case would be $x^3 = 0$. The slow convergence can easily be demonstrated by running Newton’s method using just paper and pencil.