The following assignments are due on Tuesday, March 1 at 12:30 noon. No homework will be accepted after that time. You should give me your homework in class or put it under my office door (WWH612). Do not use my mailbox in the WWH lobby.

You should turn in your MATLAB/OCTAVE programs and printouts which shows how they work for different matrices. In addition, it is important that you comment on what you can learn about the performance of the methods. Always use format long e in MATLAB. Note that in MATLAB, all numbers are represented with approximately 16 decimals.

1. A banded, square matrix $A$, with band width $k$ satisfies $a_{ij} = 0$ for all $i$ and $j$ such that $|i - j| > k$.

Suppose we wish to develop a special Gaussian elimination program to solve $Ax = b$ for banded matrices. How much storage is required if pivoting is not required? How much is needed if we allow pivoting?

2. Suppose we wish to develop a special Cholesky program to solve $Ax = b$ for banded matrices. Do you our best not to store matrix elements which are and remain zero. Note that Cholesky only works for positive definite, symmetric matrices and that you can decide by inspection if a matrix is symmetric. Whether or not the matrix is positive definite can only be decided by a computation. Test your program on at least one positive definite matrix; note that a diagonally dominant matrix is always positive definite. Also test it on at least one matrix that is not positive definite; incorporate into your program a check which tells you if the symmetric matrix is not positive definite.

3. A number of algorithms for the computation of eigenvalues of symmetric matrices begin by computing a symmetric tridiagonal matrix with the same eigenvalues. This can be done by pre- and post-multiplying the matrix by an orthogonal matrix and its transpose. Typically, this is done by using Householder matrices the first of which transforms the first column
and first row of the given matrix by introducing zeros except in the first and second position. Note that the other matrix element will also take on new values. As in Gaussian elimination, we then process an \((n-1) \times (n-1)\) matrix in the same way, and so on.