

YOUR NAME:

Numerical Analysis, Quiz # 3, with answers, April 30, 2015.

Give brief explanations of your answers.

Cross out what is not meant to be part of your answers.

1. Consider a numerical quadrature rule where $\int_{-1}^1 f(x)dx$ is approximated by

$$W_0f(x_0) + W_1f(x_1) + W_2f(x_2).$$

- (a) What is the largest family of polynomials for which this type of method can be made exact?
- (b) How do we determine the x_i so that the accuracy in this sense is as large as possible?
- (c) Consider a numerical quadrature rule for $\int_{-1}^1 \frac{1}{\sqrt{1-x^2}} f(x)dx$ of the form

$$W_0f(x_0) + W_1f(x_1) + W_2f(x_2).$$

Will $x_0, x_1,$ and x_2 be same as for the quadrature rule considered in the previous problem? If so, why? If not, why not?

- (a) All polynomials of degree 5 and less can be integrated exactly using three x_i and three weights according to the basic theorem on Gaussian quadrature. Note that that polynomial space has dimension equal to 6 and that we have 3 x_i and 3 weights at our disposal.
- (b) The x_i must be chosen as the roots of the orthogonal polynomial of degree 3 determined by the inner product

$$\int_{-1}^1 f(x)g(x)dx.$$

It is the degree 3 Legendre polynomial.

- (c) The different weight functions will produce different sets of orthogonal polynomials and their roots will be different.

2. Consider cubic spline interpolation, which leads to a linear system of equations with a number of unknowns which equal the number of interior points in the interval.

- (a) Does this system have some attractive features? Explain.
- (b) What is the origin of the system, i.e., what condition does it reflect?

- (a) The linear system has a tridiagonal, positive definite coefficient matrix and it can be solved using only a number of arithmetic operations which is linear in the number of interpolation points.
- (b) The equations of the linear system express the fact that the splines have continuous first derivatives; cf. the text book.