YOUR NAME:

Numerical Analysis, Quiz # 1, March 3, 2015, with answers.

Give brief explanations of your answers.

Cross out what is not meant to be part of your answers.

1. (a) How do we compute a new iterate x_{k+1} from x_k if we want solve a nonlinear equation $f(x) = 0$ by Newton's method?

   (b) Consider $f(x) = x \sin(\pi x)$. This function has zeros at $0, +1, -1, +2, -2,$ etc. What can we say about the rate of convergence of finding the zero of $f(x) = 0$ at 0 and the one at 1? Assume that we start the iterations close to the root.

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$$x_{k+1} = x_k - f(x_k)/(df(x_k)/dx).$$

Newton's method will converge quadratically to a root ξ provided that $df(\xi)/dx \neq 0$. Therefore we will have rapid convergence to all the roots of the given function which differ from 0. The function has a double root at 0 and Newton's method will converge much slower to that root. Compare Problem 1.6 in the textbook.

2. Linear systems of algebraic equations, $Ax = b$, where A is a square, invertible matrix of order n and x and b are $n-$vectors are solved by using *partial pivoting.* Recall that then rows of the matrix are exchanged to assure that the elements of the lower triangular matrix L all satisfy $|\ell_{ij}| \leq 1$.

   Consider the matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

   If we apply Gaussian elimination with partial pivoting of this matrix, in what order will the rows of the matrix emerge? In other words, what is the permutation matrix P? Just do the minimal computations to answer this specific question.

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   Recall that Gaussian elimination with pivoting corresponds to factoring PA into the product of a unit lower triangular matrix L and an upper triangular matrix U. The elements on the diagonal of L are all equal to 1 and those below the diagonal less than or equal to 1 in absolute value. P is a permutation matrix which potentially changes the order of the rows of A.

   In the first step, we will exchange the first and the third rows; this brings 7 into the 11 position and makes the two elements of the lower triangular matrix in the first column less than 1 in absolute value.

   We then have to compute and compare the new values of the elements in position 22 and 23. We then find that we need to exchange the second and third rows since the element in the 22 position is smaller that the one in position 23. Therefore the original row 3 ends up on top, followed by the original row 1 and then the original row 2.