Assignment Set 6, MATH-UA0252-1

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April 14, 2015

The following assignments are due on Monday April 27 at 12:00 noon. No homework will be accepted after that time. You should give me your homework in class or put it under my office door (WWH612). Do not use my mailbox in the WWH lobby.

You should turn in your MATLAB/OCTAVE programs and printouts which shows how they work in different cases. In addition, it is important that you comment on what you can learn about the performance of the methods. Always use format long e in MATLAB. Note that in MATLAB, all numbers are represented with approximately 16 decimals; we will discuss computer arithmetic during the first week of Apri.

1. Write a program that computes the natural cubic spline interpolant of any function and test it on the function of Runge's example. Try both equidistant and the alternative set of points used in an earlier homework problem.

   Estimate the error in the two cases. Is there an appreciative difference? Note that natural cubic splines have vanishing second derivatives at the end points of the interval.

   In your program, you will need a solver for tridiagonal, symmetric, positive definite systems of equations. Do not use a solver for general linear systems.

2. Modify your program to one which computes a periodic cubic spline $s_2(x)$ for a periodic function $f(x)$ with period 1. Thus $s_2(x+1) = s_2(x)$ and $f(x)$ should also satisfy $f(x+1) = f(x)$.

   What is now the structure of the linear system that must be solved and can we take good advantage of it? Try out your program for the function $\sin(\pi x)$ using many equidistant knots.

3. Write programs which computes piecewise cubic Hermite interpolation and run the same examples.

4. Run your spline programs using 1000 interpolation points. Comment on the performance.
5. Discuss and compare the quality of the approximation obtained by cubic splines, piecewise cubic Hermite, and polynomial interpolation.
6. Problem 9.1.
7. Problem 9.4.
8. Problem 9.5.
9. Problem 10.1.
10. Problem 10.2.
11. Problem 10.8.