Assignment Set 2, MATH-UA0252-1

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The following assignments are due on Friday February 27 at 12:00 noon. No homework will be accepted after that time. You should give me your homework in class or put it under my office door (WWH612). Do not use my mailbox in the WWH lobby.

You should turn in your MATLAB/OCTAVE programs and printouts which shows how they work for different functions. In addition, is is important that you comment on what you can learn about the performance of the methods; it is appropriate to count the number of function evaluations to arrive at a good approximation of the roots. An error of 10^{-6} will be appropriate for your experiments but it also makes sense to find out what maximum accuracy can be obtained. Always use format long e in MATLAB. Note that in MATLAB, all numbers are represented with approximately 16 decimals; we will discuss computer arithmetic in the middle of the term.

A banded, square matrix A, with band width k satisfies $a_{ij} = 0$ for all i and j such that $|i - j| > k$.

Suppose we wish to develop a special Gaussian elimination program to solve $Ax = b$ for banded matrices. How much storage is required if pivoting is not required? How much is needed if we allow pivoting?

1. Write a MATLAB program for Gaussian elimination of banded matrices, which uses only one loop; do so by exploiting the possibilities of working with matrices rather than with individual matrix elements.

   Make sure that you do not unnecessarily store elements that will remain zero throughout.

   If possible, also develop a version which allows for partial pivoting.

   Show, by numerical experiments, that your program works correctly by generating matrices (and right hand sides) some of which require partial pivoting, and comparing your solutions with those obtained by MATLAB and the simplest possible script.

2. A square matrix A is strictly column diagonally dominant if

$$\sum_{i \neq j} |a_{ij}| < |a_{jj}|, \quad for \quad i = 1, \ldots, n.$$

   Prove that pivoting for Gaussian elimination is not needed for a linear system with such a matrix.

   Hints: The tridiagonal case is discussed in the textbook. First prove that no pivoting is need in the first step. Then prove that the matrix that remains to be $LU-$factored is also strictly diagonally dominant.

3. Assume that a square matrix is strictly row diagonally dominant:

$$\sum_{j \neq i} |a_{ij}| < |a_{ii}|, \quad for \quad i = 1, \ldots, n.$$

   Show that any such matrix is invertible.

4. Problem 2.4 in the text book.
5. Problem 2.8 in the text book.
6. Problem 2.12 in the text book.
7. Problem 2.13 in the text book.
8. Problem 2.15 in the text book.