

Practice Final, G63.2470, Spring 2009

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Motivate your answers and cross out everything that is not part of your answers.

1. Find the solution of

$$x'' - 5x' + 4x = 4t^2e^{2t}.$$

2. Consider the solutions of

$$u''(x) + au(x)/x^2 = 0, \quad 1 \leq x < \infty.$$

Here a is a real constant. Show that if $a > 1/4$, then any solution has infinitely many zeros and that it has at most a finite number of zeros if $a < 1/4$.

3. Consider a scalar linear ordinary differential equation of second order. Assume that one solution is known, e.g., in terms of a power series. How can we then find a second independent solution? In particular, can you provide a formula for the second solution?
4. Consider the initial value problem for the linear system of ordinary differential equations

$$y_1' = -y_1 + y_2, \quad y_2' = y_1/(1 + t^2) - y_2.$$

What can you prove about the behavior of any solution as $t \rightarrow \infty$?

5. Find the solutions of $t^2x'' = (x')^2$ and $tx''' = x' - tx''$.
6. Find all solutions of $x'' + x^2 = t^2 - 2t$ by first finding a special simple solution and then the general one.
7. Consider an initial value problem $y' = f(t, y)$, and assume that $f(t, y)$ is only continuous. Does there always exist a solution and if so, is it necessarily defined for all t ?