Assignment Set 6, G63.2470, Spring 2009

Olof Widlund

April 27, 2009

The following assignments are due no later than May 7, at midnight.

1. Develop a numerical quadrature rule, similar to a Gaussian rule but using the roots of $(1-x^2)L'_n(x)$. Here $L_n(x)$ is the Legendre polynomial of degree n. Characterize the family of polynomials which is integrated exactly by this method. The interval of integration is $(-1, 1)$.

2. The Chebyshev polynomials of the first and second kind are defined by $T_n(x) = \cos(n \arccos(x))$ and $U_n(x) = T'_{n+1}(x)/(n+1)$. Show that all their roots are in $[-1, 1]$ and compute these roots.

3. Show that
$$(1-x^2)T''_n(x) - xT'_n(x) + n^2 T_n(x) = 0,$$
and that
$$(1-x^2)U''_n(x) - 3xU'_n(x) + n(n+2)U_n(x) = 0.$$

4. Show that the Legendre polynomial $L_n(x)$ satisfies
$$L_n(x) = \frac{1}{\pi}\int_0^\pi (x + \sqrt{x^2-1}\cos(\phi))^n d\phi.$$

5. Develop a power series solution of the differential equation
$$z(1-z)w'' + (1-2z)w' - w/4 = 0.$$
What are the nature of the singularities of this equation and what can be said about the existence of such series? Discuss the radius of convergence of your power series.