

Assignment Set 5, G63.2470, Spring 2009

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The following assignments are due on April 8, at midnight, but will be accepted up to one week late.

1. Problem 2 of chapter 7 of Coddington and Levinson.
2. Problem 11 of chapter 7 of Coddington and Levinson.
3. Problem 16 in chapter 7 of Coddington and Levinson.
4. The system of differential equations

$$y_1' = y_1(y_1^2 + y_2^2)^{-1/2} - y_1 - y_2, \quad y_2' = y_2(y_1^2 + y_2^2)^{-1/2} - y_2 + y_1$$

has a periodic solution $y_1(x) = \cos(x), y_2 = \sin(x)$. Study the direction fields on circles centered at the origin and show that any semiorbit will converge to a limit set, which is the unit circle.

5. Develop a criterion for deciding if a solution of a boundary value problem

$$y' = F(x, y), \quad x_1 \leq x \leq x_2, \quad B_1 y(x_1) + B_2 y(x_2) = b$$

is isolated. y is a vector valued function and B_1 and B_2 are constant square matrices. $F(x, y)$ is nonlinear.

Hint: Linearize the problem around a solution and consider a small perturbation of it. Find conditions which makes it possible to show that there is no second solution nearby. If possibly, give a full proof.