# Assignment Set 3, G63.2470, Spring 2009

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February 23, 2009

**The following assignments are due on March 4 at midnight.**

1. Given the Jordan canonical form or a real matrix, show how construct the real Jordan form. (This canonical form was used but not extensively discussed in the February 18 lecture.) Recall that for each complex conjugate pair of eigenvalues, there is Jordan block built from two-by-two submatrices; these matrices could have just one diagonal block. The off-diagonal blocks, next to and above the diagonal blocks, are all the same multiple of the two-by-two identity matrix. The diagonal matrix blocks are skew-symmetric and have the real part of the eigenvalue in their diagonal positions and itsi $+/-$ the imaginary part in the other positions.

2. Consider the linear system
$$y' = (A + B(t))y,$$
where $A$ is a constant matrix and all solutions of $y' = Ay$ go to zero when $t \to \infty$. Show that all solutions of $y' = (A + B(t))y$ are bounded if $\int_0^\infty \|B(t)\| dt < \infty$.

   Solve the following equations:

3. $2txdt + (t^2 - x^2)dx = 0$

4. $\frac{3t^2+x^2}{x^2}dt - \frac{2t^3+5x}{x^3}dx = 0$

5. $3t^2(1 + \log(x))dt = (2x - \frac{t^3}{x})dx$

6. $(t^2 + x^2 + t)dt + xdx = 0$

7. $(t^2 + 3\log(x))xdt = tdx$

8. $xdt - tdx = 2t^3 \tan(x/t)dt$

9. $x^2dt + (e^t - x)dx = 0$

10. $txdt = (x^3 + t^2x + t^2)dx$