Problem 1
Find ALL of the EXACT solutions to the equation
\[ \sin(\theta) = \frac{\sqrt{3}}{2} \]

Problem 2
The front door to the student union is 20 feet above the ground, and it is reached by a flight of step. The school wants to build a wheel-chair ramp, with an incline of 15°, from the ground to the door. How much horizontal distance is needed for the ramp.

Problem 3
Let \( \Delta \) be a triangle with angles \( \alpha, \beta, \gamma \) with opposite sides \( a, b, c \) respectively. If \( a = 7, \ b = 6, \ \gamma = 12^\circ \), find \( c, \alpha, \) and \( \beta \). HINT: Recall the Law of Sine and Cosine
\[
\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{c} \quad \text{and} \quad c^2 = a^2 + b^2 - 2ab \cos(\gamma)
\]

Problem 4
Find the EXACT value of \( \sin(345^\circ) + \cos(165^\circ) \). HINT: Recall the Angular Sum-Difference Formula
\[
\cos(A + B) = \cos(A) \cos(B) - \sin(A) \sin(B) \quad \cos(A - B) = \cos(A) \cos(B) + \sin(A) \sin(B) \\
\sin(A + B) = \sin(A) \cos(B) + \cos(A) \sin(B) \quad \sin(A - B) = \sin(A) \cos(B) - \cos(A) \sin(B)
\]
Problem 5

a) Convert the Cartesian coordinate \((\sqrt{3}, 1)\) to polar coordinates. Give the EXACT answer.
b) Convert the polar coordinate \((2\sqrt{3}, 5\pi/6)\) to Cartesian coordinates. Give the EXACT answer.

Problem 6

Given the complex number \(z = (2 - 2i)^{2/3}\), express \(z\) in Cartesian form \(z = x + iy\).

Problem 7

Given \(f(x) = \frac{x^2 + x - 1}{x^2 - 2x - 3}\)

a) What are the EXACT values of the \(x\)-intercepts (if any)?
b) What are the EXACT values of the \(y\)-intercepts (if any)?
c) What are the vertical asymptotes (if any)?
d) What is the horizontal asymptote (if any)?
e) sketch the graph of \(f(x)\).

Problem 8

Find the 5th degree polynomial \(p(x)\) such that \(p\) has a double zero at \(x = 2\), \(p(-5) = p(-1) = p(4) = 0\), and \(p(0) = 2\). Then graph it.

Problem 9

Find the exact value of \(\sum_{k=0}^{11} 3 \left(\frac{1}{4}\right)^k\). Your answer should be fully simplified.

Problem 10

Given the recurrence relation \(a_n = 4a_{n-1}; a_1 = 2\), find the formula for the general term \(a_n\) as a function of \(n\).
Problem 11

Express each of the following complex numbers in Cartesian form $x + iy$

a) $(0.5 + 0.3i) - (1.9 - 2i)$

b) $(2 + 3i)(4 - i)$

c) $\frac{-2-4i}{5+3i}$

Problem 12

Prove the identity

$$\frac{1 + \sin \theta}{\cos \theta} = \frac{\cos \theta}{1 - \sin \theta}$$

HINT: Recall the identity $\sin^2 \theta + \cos^2 \theta = 1.$