No books or calculators are permitted, however you may use one 8.5 × 11 sheet of notes.

(1) (15 points) Let $x(t)$, $v(t)$, and $a(t)$ be the position, velocity, and acceleration of a body at time $t$. Suppose $a(t) = t$, $v(0) = 1$, and $x(0) = 0$.

(a) Find $v(t)$.
(b) Find $x(t)$.
(c) Find the total distance travelled during the time $0 \leq t \leq 1$.

(2) (25 points) Evaluate the following indefinite integrals. Briefly justify your steps.

(a) $\int (x^2 - 3x)(2x - 3)\,dx$
(b) $\int \sin^3 x \cos x \,dx$
(c) $\int \frac{x^3}{e^{x^2}} \,dx$
(d) $\int x(3x - 5)^{11} \,dx$
(e) $\int \tan 2x \,dx$

(3) (15 points) Evaluate the following definite integrals. Briefly justify your steps.

(a) $\int_0^{\pi/2} \cos x \,dx$
(b) $\int_0^1 \frac{x}{3-x^2} \,dx$
(c) $\int_0^1 e^{-2x} \,dx$

(4) (10 points) Consider the two functions

$$F(x) = \int_0^x \frac{\sin t}{1 + t^2} \,dt \quad \text{and} \quad G(x) = \int_0^x \frac{\sin t}{1 + t^2} \,dt$$
on the interval $0 \leq x \leq 2\pi$.

(a) Find $F'(x)$, and identify the critical numbers of $F$ (i.e. solutions of $F'(x) = 0$ satisfying $0 \leq x \leq 2\pi$).
(b) Find $G'(x)$, and identify the critical numbers of $G$. 

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(5) (20 points) Let \( f(x) = 6 - x^2 \).
(a) Sketch the region bounded by \( f(x) \) and the \( x \)-axis.
(b) Find the area of this region.
(c) Find the volume of the body obtained by revolving the region about the \( x \)-axis.
(d) Find the volume of the body obtained by revolving the region about the \( y \)-axis.

(6) (15 points) Consider the region between the graphs of \( y = x + 1 \) and \( y = 1 - x^2 \), for \( 0 < x < 1 \).
(a) Sketch this region.
(b) Find its area, using integration in \( x \).
(c) Explain how you could alternatively have found its area using integration in \( y \) (to save time, you need not evaluate the integrals).

(7) (15 points) Let’s consider inverse functions.
(a) We learned that if \( f \) is a differentiable function which is one-to-one, then its inverse \( f^{-1} \) is differentiable and
\[
(f^{-1})'(y) = \frac{1}{f'(f^{-1}(y))}
\]
provided that \( f'(f^{-1}(y)) \neq 0 \). Explain why this formula is valid, starting from the identity \( f(f^{-1}(y)) = y \) and assuming that \( f^{-1} \) is differentiable.
(b) Now consider \( f(x) = x^3 + 2x - 5 \). Notice that \( f(1) = -2 \).
   (i) Show that \( f \) is one-to-one.
   (ii) Calculate \( (f^{-1})'(-2) \).

(8) (10 points) If a function \( f \) satisfies \( f > 0 \) and \( f' > 0 \), then
\[
\frac{1}{4}[f(0) + f(1/4) + f(1/2) + f(3/4)] \leq \int_0^1 f(x) \, dx \leq \frac{1}{4}[f(1/4) + f(1/2) + f(3/4) + f(1)].
\]
Explain why this is true, using the relationship between lower sums, upper sums, and integrals.