Calculus I Midterm Exam through Section 3.5
September 28, 2005

No books or calculators are permitted, however you may use one 8.5 \times 11 sheet of notes. There are 17 problems, worth 10 points each, for a total possible of 170 points.

Part I: Limits

Problems 1-3 address the function \( f(x) \) whose graph is shown in the figure.

1) Evaluate each limit; if it doesn’t exist, say briefly why not:
   (a) \( \lim_{x \to 2^-} f(x) \)  
   (b) \( \lim_{x \to 2^+} f(x) \)  
   (c) \( \lim_{x \to 2} f(x) \)
2) Evaluate each limit; if it doesn’t exist, say briefly why not:
   (a) \( \lim_{x \to 4^-} f(x) \)  
   (b) \( \lim_{x \to 4^+} f(x) \)  
   (c) \( \lim_{x \to 4} f(x) \)
3) Some further questions about \( f \):
   (a) What is \( f(2) \)? Is \( f \) continuous at \( x = 2 \)?
   (b) What is \( f(4) \)? Is \( f \) continuous at \( x = 4 \)?

4) Find the limit, or explain why it doesn’t exist:
   (a) \( \lim_{x \to 5} \sqrt{x + 4} \)  
   (b) \( \lim_{x \to 0} \frac{x}{|x|} \)

5) Find the following limit, or explain why it doesn’t exist: \( \lim_{x \to 4} \frac{x^2 + 2x - 24}{x - 4} \)

6) Find the following limit, or explain why it doesn’t exist: \( \lim_{h \to 0} \frac{1 - \frac{1}{x^2}}{1 - \frac{h}{x}} \)

7) Explain, by taking the limit of the difference quotient \( (f(x + h) - f(x))/h \) as \( h \to 0 \), why the derivative of \( f(x) = x^2 \) is \( f'(x) = 2x \).

8) Same as question (7), but this time explain why the derivative of \( f(x) = 1/x \) is \( f'(x) = -1/x^2 \).
Part II: Differentiation

In Parts II and III you can (and should) use the standard rules for differentiation. Any correct answer is OK (algebraic simplification is not required).

9) Find $f'(x)$ when
   
   (a) $f(x) = 3x^2 + 1$  
   (b) $f(x) = (x^2 + 5)(x - 7)$  
   (c) $f(x) = \frac{x^3 + 5x^2 - 8}{x^2 + 8}$

10) Find $dy/dx$ when
    
    (a) $y = (x^2 + 2x)^3$  
    (b) $y = \sqrt{x^2 - 1}$

11) Find $f'(x)$ when
    
    (a) $f(x) = \sqrt{x(x - 2)}$  
    (b) $f(x) = \frac{\sqrt{x(x - 2)}}{x^2 + 2}$

12) Evaluate the second derivative $f''(x)$ when
    
    (a) $f(x) = x^{10}$  
    (b) $f(x) = \sqrt{2x + 1}$

13) Evaluate
    
    (a) $\frac{d}{dx} \left( \frac{x^4 + 2}{x} \right)$  
    (b) $\frac{d}{dx} \left[ \frac{1}{x^2} \frac{d}{dx} \left( \frac{x^4 + 2}{x} \right) \right]$  

Part III: Some uses of differentiation.

14) Consider the function $f(x) = 1/x$.
    
    (a) What is $f'(x)$?
    (b) Find the equation of the line tangent to the graph of $f$ at $x = 1$.
    (c) At which $x$ does the line normal to the graph have slope 2?

15) Consider the function $f(x) = x^3 - 3x + 1$.
    
    (a) What is $f'(x)$?
    (b) When is the line tangent to the graph of $f$ horizontal?
    (c) For which $x$ is $f$ increasing?

16) You throw a ball up in the air, with initial velocity 16ft/sec. We know from Section 3.4 that the ball’s height about the ground at time $t$ is $h(t) = -16t^2 + 16t$.
    
    (a) At what time $t$ does the ball reach its maximum height?
    (b) At what time $t$ does the ball return to the ground?

17) Consider the function $f(x) = \begin{cases} x^2 & x < 1 \\ 4x - 3 & x \geq 1 \end{cases}$
    
    (a) Is $f$ continuous at $x = 1$? Why or why not?
    (b) Is $f$ differentiable at $x = 1$? Why or why not?