Calculus I – HW Solutions – problems from § 4.2

Section 4.2

(6) \(f(x) = x(x+1)(x+2) \Rightarrow f'(x) = 5(x+1)(x+2) + x(x+2) + x(x+1)\)
\[= 3x^2 + 6x + 2\]

The roots of \(f'(x) = 0\) are \[x = \frac{-6 \pm \sqrt{12}}{6} = -1 \pm \frac{1}{\sqrt{3}}\]

Clearly \(f'(x) > 0\) as \(x \to \pm \infty\)

\[f'(1) = -1 < 0\]

So \(f'(x) > 0\) (\(f\) is increasing) \(\Rightarrow x < -1 - \frac{1}{\sqrt{3}}\)

\[f'(x) < 0\] (\(f\) is decreasing) \(\Rightarrow -1 - \frac{1}{\sqrt{3}} < x < -1 + \frac{1}{\sqrt{3}}\)

\[f'(x) > 0\] (\(f\) is increasing) \(\Rightarrow x > -1 + \frac{1}{\sqrt{3}}\)

(8) \(f(x) = 2x - \frac{1}{2}x^2 \Rightarrow f'(x) = 2 - x^3\)
The derivative vanishes when \(x^3 = -1\), i.e. \(x = -1\); it's undefined (thus, not continuous) at \(x = 0\).

Clearly \(f' > 0\) for \(x \to -\infty\)

\[f' \to \infty\] as \(x \to 0\) from above

\[f' \to -\infty\] as \(x \to 0\) from below

So \(f'(x) > 0\) (\(f\) is increasing) \(\Rightarrow x < -1\)

\[f'(x) < 0\] (\(f\) is decreasing) \(\Rightarrow -1 < x < 0\)

\[f'(x) > 0\] (\(f\) is increasing) \(\Rightarrow x > 0\)

(22) \(f(x) = e^{x^2} \Rightarrow f'(x) = 2xe^{x^2}\)

on \(0 \leq x \leq \pi\), \(f'\) vanishes exactly at \(x = 0\), \(x = \frac{\pi}{2}\), and \(x = \pi\).

Clearly \(f'(x) < 0\) for \(0 < x < \frac{\pi}{2}\), \(f'(x) > 0\) for \(\frac{\pi}{2} < x < \pi\), so \(f'(x) < 0\) (\(f\) is decreasing) \(\Rightarrow 0 < x < \frac{\pi}{2}\)

\[f'(x) > 0\] (\(f\) is increasing) \(\Rightarrow \frac{\pi}{2} < x < \pi\)
(26) \( f'(x) = 2x - 5 \) and \( f(2) = 4 \) \( \Rightarrow \) \( f(x) = x^2 - 5x + \text{const} \)
Since \( f(2) = 4 \) we have \( 4 = 4 - 5 \cdot 4 + \text{const} \) \( \Rightarrow \) \( \text{const} = 20 \)
Thus \( f(x) = x^2 - 5x + 20 \)

(32) \( f'(x) = 4x + \cos x \) \( \Rightarrow \) \( f(x) = 2x^2 + \sin x + \text{const} \)
We're told \( f(0) = 1 \) \( \Rightarrow \) \( 1 = 0 + 0 + \text{const} \) \( \Rightarrow \) \( \text{const} = 1 \)
Thus \( f(x) = 2x^2 + \sin x + 1 \).

(42) Looking at the graph of \( f' \):
- \( f'(x) = 0 \) for \( x < -2 \), with a jump discontinuity at \( x = -2 \)
- \( f'(x) < 0 \) for \(-2 < x < 1\)
- \( f'(x) > 0 \) for \(-1 < x < 1\), and \( f' \) is largest at \( x = 0 \)
- \( f'(x) < 0 \) for \( 1 < x < 2 \), with a jump discontinuity at \( x = 2 \)
- \( f'(x) > 0 \) for \( x > 2 \) (wherever it's value is constant)

So the graph of \( f' \) looks like this:

![Graph of f']

(62) We use the fact that \( \sin x < x \) for all \( x > 0 \).
(For \( 0 < x < \pi/2 \) this is clear from basic geometry, see Sec 2.5 exp 72, 104; for \( x > \pi/2 \) it's clear since \( \sin x < 1 \).)
Let \( f(x) = 1 - x/2 \), \( g(x) = \cos x \); Then \( g - f \) vanishes at \( x = 0 \) and \((g - f)' = g' - f' = x - \sin x > 0\). So \( g - f \) is an increasing function of \( x \) for \( x > 0 \), and \( g(\pi) - f(\pi) > g(0) - f(0) = 0 \).
Thus \( g(x) > f(x) \), asdesired.