Section 2.4

(2) \( g \) is cont's \( f \) \( -4 < x < 1, \ -1 < x \leq 3, \ 3 < x \leq 5, \ 5 < x \leq 8 \)

(4) \( g(x) = \sqrt{(x-1)^2 + 5} \) is cont's at \( x = 1 \) since \( g \) is the composition of continuous functions.

(10) \( g(x) = \begin{cases} 
    x^2 + 5 & x < 2 \\
    10 & x = 2 \\
    x^3 + 1 & x > 2 
\end{cases} \) is not continuous at \( x = 2 \); it has a pt discontinuity.

\[
\lim_{{x \to 2^-}} g(x) = \lim_{{x \to 2^+}} g(x) = 9 \quad \text{but} \quad g(2) = 10
\]

(28) \( g(x) = \begin{cases} 
    x + 7 & x < -3 \\
    12 - 2x & -3 \leq x < -1 \\
    x^2 - 2x & -1 \leq x < 3 \\
    2x - 3 & 3 \leq x
\end{cases} \)

- has a jump discontinuity at \( x = -3 \);
- has a removable discontinuity at \( x = -1 \);
- it's cont's at \( x = 3 \)

(50) Many answers are possible. Here's one:

(55) We need \( A(x-3) = x^2 \) at \( x = 1 \), so we used \( A = 1 \).
Section 3.1

2. The difference quotient is \[ \frac{[7(x+h) - (x+h)^2] - [7x - x^2]}{h} \].

This simplifies to \( 7 - 2x \). The limit as \( h \to 0 \) is \( 7 - 2x \). So \( f'(x) = 7 - 2x \). At \( x = 2 \) we get \( f' = 3 \).

6. The difference quotient is \[ \frac{\sqrt{6-(x+h)} - \sqrt{6-x}}{h} \].

This simplifies to \( \frac{-1}{\sqrt{6-x+h} + \sqrt{6-x}} \). The limit as \( h \to 0 \) is \( \frac{-1}{2\sqrt{6-x}} \). At \( x = 2 \) we get \( f' = -\frac{1}{4} \).

8. The difference quotient is 0 (for any \( x \) and \( h \)). So \( f' = 0 \).

12. The difference quotient is \( \frac{1}{3} \left( \frac{1}{x+h+3} - \frac{1}{x+3} \right) \).

This simplifies to \( \frac{-1}{(x+3)(x+3+h)} \). The limit as \( h \to 0 \) is \( \frac{-1}{(x+3)^2} \).

18. \( f(x) = \frac{1}{2} x^{-1/2} \) so \( f'(4) = \frac{1}{4} \). Note: \( f(4) = 2 \)

Tangent line: \( \frac{y-2}{x-4} = \frac{1}{4} \); normal line: \( \frac{y-2}{x-4} = -4 \)

20. \( f'(2) = -3(2)^2 = -12 \), \( f(2) = 5 - 2^3 = -3 \). So

Tangent line: \( \frac{y+3}{x-2} = -12 \); normal line: \( \frac{y+3}{x-2} = \frac{1}{12} \)

22. There's a jump discontinuity at \( x = 2 \), \( g \) is continuous but not differentiable at \( x = -2 \) and \( x = 3 \).
Section 3.2

(2) $F'(x) = -6x^3$ using the power rule

Section 3.2

(2) $F'(x) = 2$

(4) $F' = -6x^{-3}$

(6) $F'(x) = \frac{2x(x^3) - 3x^2(x^2+2)}{(x^3)^2} = \frac{-1}{x^2} - \frac{6}{x^4}$

(24) $F'(x) = \frac{-(x^2 + x + 1)(2x+2) + (4x+1)(x^2+2x+1)}{(x^2+2x+1)^2}$

At $x = 0$ we get $F'(0) = \frac{-2 + 1}{1} = -1$

At $x = 1$ we get $F'(1) = \frac{-4 + 4 + 5.4}{16} = \frac{1}{4}$

(28) $F'(x) = 3(2x f(x) + x^2 f'(x)) = 5$

So if $f(0) = 3, f'(0) = 2$ then $F'(0) = -5$.

Actually we don't need to know $f(0)$ or $f'(0)$ to see this.

(36) $F(x) = 2x + \frac{16}{x^2}$. Tangent line is horizontal when $F'(0) = 0$, i.e. $2x + \frac{16}{x^2} = 0$. This simplifies to $x^3 = -8$, so $x = -2$

(44) Lines are parallel if their slopes satisfy $m_1 = m_2$. Here $m_1 = \text{slope of 1st line} = F'(x) = 3x^2 - 3$

$m_2 = \text{slope of 2nd line} = \frac{3}{5}$. We want $\frac{3}{5}(3x^2 - 3) = -1$

Solve for $x = \pm 2/3$. 

continuously but not differentiable at $x = 2$

slope jumps from 4 to 0