Calculus I Midterm taken 9/28/05 - Solutions

1. (a) \( \lim_{x \to 2^-} f(x) = -\infty \)  
   (b) \( \lim_{x \to 2^+} f(x) = -\infty \)  
   (c) \( \lim_{x \to 2} f(x) = -\infty \)  

2. (a) \( \lim_{x \to 4^-} f(x) = 3 \)  
   (b) \( \lim_{x \to 4^+} f(x) = 2 \)  
   (d) \( \lim_{x \to 4} f(x) \) does not exist  

3. (a) \( f(2) \) is undefined; therefore \( f \) is not continuous at \( x = 2 \)  
   (b) \( f(4) = 3 \); \( f \) is discontinuous at \( x = 4 \) (it has a jump discontinuity)  

4. (a) \( f(9) = 3 \)  
   (b) left limit is \(-1\), right limit is \(+1\), 
   so the limit as \( x \to 0 \) does not exist  

5. \( (x^2 + 2x - 24) = (x+6)(x-4) \)  
   no limit as \( \lim_{x \to 4} (x+6) = 10 \)  

6. \( \frac{1 - \frac{1}{h^2}}{1 - \frac{1}{h^3}} = \frac{h^3 - h}{h^3 - 1} \)  
   so limit as \( h \to 0 \) is \( \frac{0}{-1} = 0 \)  

7. \( \frac{(x+h)^2 - x^2}{h} = 2xh + h^2 = 2x + h \)  
   so limit as \( h \to 0 \) is \( 2x \)  

8. \( \frac{1}{x+h} - \frac{1}{x} = \frac{x - (x+h)}{x(x+h)} = -\frac{1}{x(x+h)} \)  
   so limit as \( h \to 0 \) is \( -\frac{1}{x^2} \)  

9. (a) \( f' = 6x \)  
   (b) \( f' = (x^2 + 5)(1) + 2x(x-7) \)  
   (c) \( f' = \frac{(3x^2 + 10x)(x^2 + 8) - (x^3 + 5x^2 - 8)(2x)}{(x^2 + 8)^2} \)
(10) a) \[ \frac{dy}{dx} = 3(x^2 + 2x)^2 \cdot (2x + 2) \]

b) \[ \frac{dy}{dx} = \frac{1}{2}(x^2 - 1) \cdot 2x \]

(11) a) \[ f' = \frac{3}{2} x^{1/2} - x^{-1/2} \]

b) \[ f' = \frac{x^{-1/2}}{(x^2 - 1) \cdot (2x + 2) - 2x \cdot (x^{2/2} - 2x^{1/2})}{(x^2 + 2)^2} \]

(12) a) \[ f' = 10x^6 \Rightarrow f'' = 90x^5 \]

b) \[ f' = \frac{1}{z} (2x + 1)^{-1/2} \Rightarrow f'' = \frac{-1}{2} (2x + 1)^{-3/2} \]

(13) a) \[ \frac{d}{dx} \left( x^2 + \frac{2}{x} \right) = 3x^2 - 2x^{-2} \]

b) \[ \frac{d}{dx} \left[ \frac{3x^2 - 2x^{-2}}{x^2} \right] = \frac{d}{dx} \left[ 3 - 2x^{-4} \right] = 8x^{-5} \]

(14) a) \[ f'(x) = -\frac{1}{x^2} \]

b) \[ x_0 = 1, f(x_0) = 1, \text{ slope is } f'(x_0) = -1 \Rightarrow \text{ line is } y = \frac{x - 1}{x - 1} = -1 \]

c) \[ \text{normal line has slope } -\frac{1}{f'(x)} = x^2. \text{ We want this to equal } 2. \text{ Solution is } x^2 = 2 \Rightarrow x = \pm \sqrt{2} \]

(15) a) \[ f' = 3x^2 - 3 \]

b) \[ f'(x) = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1 \]

c) \[ f'(x) > 0 \Rightarrow x^2 > 1 \Rightarrow x > 1 \text{ or } x < -1 \]

(16) a) \[ f'' = 0 \Rightarrow \frac{-32t + 16}{0} \Rightarrow t = \frac{1}{2} \text{ sec} \]

b) \[ t = 0 \Rightarrow -16t^2 + 16t = 0 \Rightarrow t = 0 \text{ (initial time)} \text{ or } t = 1 \text{ sec} \]

(17) a) \[ \text{at } x=1, \quad x^2 = 1 \text{ and } 4x - 3 = 1. \text{ They catch } \Rightarrow f \text{ is cont's} \]

b) \[ \text{at } x=1, \quad \text{slopes from right & left are } \frac{dy}{dx} \left[ 4x - 3 \right] = 4 \text{ and } \frac{dy}{dx} \left[ x^2 \right] = 2x = 2. \text{ They don't catch } \Rightarrow \text{ f is not differentiable} \]