1. [20 points] Consider the solid bounded by the cylinder

\[ x^2 + y^2 = a^2 \]

the \( xy \)-plane, and the paraboloid

\[ z = x^2 + y^2 + 1 \]

Compute the flux of the vector field

\[ x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \]

through the boundary of the surface.

2. [20 points] Given \( a,b,c > 0 \), compute the volume of the tetrahedron defined by the four points \((0,0,0),(a,0,0),(0,b,0),(0,c,0)\). (HINT: the plane \( \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \) is relevant to the problem)

3. [20 points] Find the area of the surface given by

\[ z = xy \]

for \( x^2 + y^2 \leq 1, \ x \geq 0, \ y \geq 0 \).

4. [20 points] Minimize \( xy^2 \) on the circle \( x^2 + y^2 = 1 \). Use two different methods:

(a) Method of Lagrange multipliers

(b) Parameterize constraint set and use one-variable calculus
5. [20 points] Evaluate the line integral of $e^x \sin y \, i + e^x \cos y \, j$

(a) Around the circle $C : (x - a)^2 + (y - b)^2 = d^2$.
(b) Around the half-circle $C : x^2 + y^2 = 1, \ y > 0$