Motivation and Previous Work

We wish to model the alongshore current forced by obliquely incident breaking waves. A classic theory for alongshore currents is due to Longuet-Higgins; he used the radiation-stress theory for ocean waves to compute the effective mean force exerted by breaking waves, which balanced against frictional forces will yield a steady alongshore current.

One consequence of this theory is that the cross-shore location of largest current coincides with the strongest wave breaking. However, experimentally this is not true for barred beaches. Instead, alongshore currents on barred beaches maximum in the trough; that is, at a location of little or no wave breaking.

We propose that the vorticity dynamics produced by inhomogeneous wave breaking (spatially localized packets of waves) can explain the observed dislocation of current.

This was studied theoretically and numerically by Buhler and Jacobson (JFM 2001) using a wave-resolving shallow water model. Numerical results confirm the hypothesis (see right). We wish to point out two theoretical points from this paper: BJ argue that slow evolution obeys the rigid-upper-lid dynamics

\[ \nabla \cdot (\hat{u}u) = 0 \]
\[ \frac{D\hat{u}}{Dt} = -\frac{1}{\rho} \nabla \times \hat{u} \times \hat{u} \]

and the non-rotational contribution to the radiation stress convergence term can be written

\[ \frac{1}{h} \nabla \cdot \hat{S} - I_r \nabla \cdot \hat{S} = \mathbf{F} = \frac{k}{h} \nabla \cdot (k \mathbf{F}) \]

Resolving surface waves forces a short time step to be used, limiting the time scale at which this effect can be studied. In addition the self-induced shock formation of waves severely limits the amplitude of incoming waves that may be used. We propose to extend this study by constructing a model that works at the level of the slow dynamics. We will use a realistic model of wave evolution and breaking in order to compute the impact of wave dissipation on the flow.

We solve the shallow water equations with a rigid lid in the vorticity-stream formulation:

\[ \frac{\nabla \cdot (\hat{u}u)}{h} = hq \]
\[ \frac{Dq}{Dt} = \frac{q'}{h} \nabla \times \left( \frac{\hat{u}u}{h} \right) - \frac{1}{h} \nabla \times \mathbf{F} \]

on the rectangular domain \(0 < x < D, \ 0 < y < L\).

The right-hand side of the equations contain a dissipative forcing due to wave breaking and a quadratic bottom friction term.

Boundary conditions must be specified on the stream function. They are periodic in \(y\), no normal flow at \(x=0\), and \textbf{exact} on the seaward boundary \(x=D\).

The advection term is discretized using the Arakawa Jacobian. The time-stepping uses a filtered leapfrog method.

New PV-based Numerical Model

Because surface waves are not resolved, we must compute the dissipative forcing term \(\mathbf{F}\) explicitly. If we assume that the wave field is steady and does not interact with the current, we can use kinematic wave theory. Frequency is conserved along wave trajectories. Frequency is preserved along wave trajectories given by the following dispersion relation:

\[ \omega = \Omega(k,x) \]
\[ \frac{A}{dt} = \Omega_k \]
\[ \frac{A}{dt} = -\omega_x \]

In the absence of dissipation, wave action per unit area (equivocally in this case, the wave energy per unit area) satisfies the following evolution equation along wave trajectories:

\[ \frac{\partial A}{\partial t} + \nabla \cdot (c_A A) = 0 \]