1. A culture of E. coli bacteria with a good supply of nutrients doubles in size every four hours. If we start with $N = 12000$ bacteria, determine the formula for the population size $N(t)$ after $t$ hours. How many bacteria are there after one day (24 hours)? After 11 days?

2. If the cost per week of manufacturing an inexpensive transistorized audio amplifier is given by

$$C(q) = 60000 + 100q - 0.05q^2 \quad (1)$$

where $q$ is the number of units produced per week, find the cost of producing (i) 450 units per week and (ii) 600 units per week. What is the average rate of change $\Delta C/\Delta q$ of $C$ with respect to $q$ if production is raised from $q_1 = 450$ to $q_2 = 600$ units per week?

Solutions:

1. The time it takes for the population to double, i.e. the doubling time, is 4 hours. With an initial bacteria count of 12000 the formula for the population size is

$$N(t) = 12000 * 2^{t/4}$$

Therefore $N(24) = 12000 * 2^{24/4} = 12000 * 2^6 = 768000$. Also 11 days is the same as $24 * 11 = 264$ hours, so $N(264) = 12000 * 2^{264/4} = 12000 * 2^{66}$, a number which is far larger than your calculator can handle. To give an idea of its size, the number has 23 digits in it. In such a situation it’s perfectly acceptable to leave the answer in symbolic form, without evaluating the actual number.

2. 

(i) $C(450) = 60000 + 100 * 450 - 0.05 * 450 * 450 = 94875$

(ii) $C(600) = 60000 + 100 * 600 - 0.05 * 600^2 = 102000$

Thus increasing production from 450 to 600 (which means $\Delta q = 150$) causes the cost per week to increase (i.e. $\Delta C = 7125$). So the average change in cost per extra unit produced is therefore

$$\frac{\Delta C}{\Delta q} = \frac{7125}{150} = 47.5$$