Petit Dejeuner de la Finance
Paris, Nov 27, 2002

Empirical Aspects of Dispersion Trading in U.S. Equity Markets

Marco Avellaneda
Courant Institute of Mathematical Sciences, New York University
& Gargoyle Strategic Investments

What is Dispersion Trading?

- Sell index option, buy options on index components ("sell correlation")
- Buy index option, sell options on index components ("buy correlation")

Motivation: to profit from price differences in volatility markets using index options and options on individual stocks

Opportunities: Market segmentation, temporary shifts in correlations between assets, idiosyncratic news on individual stocks
Index Arbitrage versus Dispersion Trading

Index Arbitrage:
Reconstruct an index product (ETF) using the component stocks

Dispersion Trading:
Reconstruct an index option using options on the component stocks

Main U.S. indices and sectors

- Major Indices: SPX, DJX, NDX
  SPY, DIA, QQQ (Exchange-Traded Funds)

- Sector Indices:
  Semiconductors: SMH, SOX
  Biotech: BBH, BTK
  Pharmaceuticals: PPH, DRG
  Financials: BXX, XBD, XLF, RKH
  Oil & Gas: XNG, XOI, OSX
  High Tech, WWW, Boxes: MSH, HHH, XBD, XCI
  Retail: RTH
**NASDAQ-100 Index (NDX) and ETF (QQQ)**

- QQQ ~ 1/40 * NDX
- Capitalization-weighted
- QQQ trades as a stock
- QQQ options: largest daily traded volume in U.S.

**Sector Exchange Traded Funds**

- ~ 20 - 40 stocks in same sector
- Weightings by:
  - capitalization
  - equal-dollar
  - equal-stock
### Index Option Arbitrage
(Dispersion Trading)

- Takes advantage of differences in implied volatilities of index options and implied volatilities of individual stock options
- Main source of arbitrage: correlations between asset prices vary with time due to corporate events, earnings, and "macro" shocks
- Full or partial index reconstruction

### The trade in pictures

![Diagram of the trade](image)

**Sell index call**

**Buy calls on different stocks.**

Also, buy index/sell stocks
Profit-loss scenarios for a dispersion trade in a single day

Scenario 1

- Stock P/L: - 2.30
- Index P/L: - 0.01
- Total P/L: - 2.41

Scenario 2

- Stock P/L: +9.41
- Index P/L: - 0.22
- Total P/L: +9.18

First approximation to hedging: 
``Intrinsic Value Hedge’’

\[
I = \sum_{i=1}^{M} w_i S_i \quad w_i = \text{number of shares, scaled by `divisor'}
\]

\[
K = \sum_{j=1}^{M} w_j K_j \quad \Rightarrow \quad \max(I - K, 0) \leq \sum_{j=1}^{M} w_j \max(S_j - K_j, 0)
\]

\[
C_i(I, K, T) \leq \sum_{j=1}^{M} w_j C_i(S_j, K_j, T)
\]

IVH: use index weights for option hedge

IVH: premium from index is less than premium from components “Super-replication”

Makes sense for deep-in-the-money options
Intrinsic-Value Hedging is `exact' only if stocks are perfectly correlated

\[ I(T) = \sum_{i=1}^{M} w_i S_i(T) = \sum_{i=1}^{M} w_i F_i e^{\sigma N_i \frac{1}{2} \sigma^2 T} \]

\[ \rho_{ij} \equiv 1 \Rightarrow N_i \equiv N = \text{standardized normal} \]

Solve for \( X \) in:

\[ K = \sum_{i=1}^{M} w_i F_i e^{\sigma X \frac{1}{2} \sigma^2 T} \]

Set:

\[ K_i = F_i e^{\sigma X \frac{1}{2} \sigma^2 T} \]

\[ \therefore \max(I(T) - K_i, 0) = \sum_{i=1}^{M} w_i \max(S_i(T) - K_i, 0) \quad \forall T \]

Similar to Jamshidian (1989) for pricing bond options in 1-factor model

IVH : Hedge with ``equal-delta’’ options

\[ K_i = F_i e^{\sigma X \sqrt{T} \frac{1}{2} \sigma^2 T} \]

\[ \therefore X = \frac{1}{\sigma \sqrt{T}} \ln \left( \frac{K_i}{F_i} \right) + \frac{1}{2} \sigma \sqrt{T} \]

\[ -X = \frac{1}{\sigma \sqrt{T}} \ln \left( \frac{F_i}{K_i} \right) - \frac{1}{2} \sigma \sqrt{T} = d_2 \]

\[ N(d_2) = \text{constant} \]

log - moneyness = constant

Deltas = constant
What happens after you enter a trade:
Risk/return in hedged option trading

Unhedged call option

Hedged option

Profit-loss for a hedged single option position (Black–Scholes)

\[ P/L = \theta \cdot (n^2 - 1) + NV \cdot \frac{d\sigma}{\sigma} \]

\[ \theta = \text{time-decay (dollars)}, \quad n = \frac{\Delta S}{S\sigma\sqrt{\Delta t}}, \quad NV = \text{normalized Vega} = \sigma \frac{\partial C}{\partial \sigma} \]

\( n \sim \text{standardized move} \)

\[ \text{Gamma P/L for an Index Option} \]

Assume \( d\sigma = 0 \)

\[ \text{Index Gamma P/L} = \theta_i (n_i^2 - 1) \]

\[ n_i = \sum_{j=1}^{M} \frac{p_j \sigma_j \rho_{ij}}{\sigma_i} \]

\[ p_i = \frac{w_i S_i}{\sum_{j=1}^{M} w_j S_j} \]

\[ \sigma_i^2 = \sum_{j=1}^{M} p_j \sigma_j \rho_{ij} \]

\[ \text{Index P/L} = \theta_i \sum_{j=1}^{M} \frac{p_j^2 \sigma_j^2}{\sigma_i^2} (n_i^2 - 1) + \theta_i \sum_{j=1}^{M} \frac{p_j \rho_{ij}}{\sigma_i^2} (n_i n_j - \rho_{ij}) \]
Gamma P/L for Dispersion Trade

$i^{th}$ stock P/L $\approx \theta_i \cdot (n_i^2 - 1)$

Dispersion Trade P/L $\approx \sum_{i=1}^{n} \left( \theta_i + \frac{p_i \sigma_i^2}{\sigma_i^2} \theta_i \right) (n_i^2 - 1) + \theta_f \sum_{i=1}^{N} \frac{p_i \sigma_i^2}{\sigma_i^2} (n_i n_f - \rho_f)$

- diagonal term: realized single-stock movements vs. implied volatilities
- off-diagonal term: realized cross-market movements vs. implied correlation

Introducing the Dispersion Statistic

$$D^2 = \sum_{i=1}^{N} p_i (X_i - Y)^2$$
$$X_i = \frac{\Delta S_i}{S_i}, \quad Y = \frac{\Delta I}{I}$$

$$D^2 = \sum_{i=1}^{N} p_i \sigma_i^2 n_i^2 - \sigma_f^2 n_f^2$$

$$\text{P/L} = \sum_{i=1}^{N} \theta_i (n_i^2 - 1) + \theta_f (n_f^2 - 1)$$
$$= \sum_{i=1}^{N} \theta_i n_i^2 + \theta_f n_f^2 - \Theta$$
$$\Theta \equiv \sum_{i=1}^{N} \theta_i + \theta_f$$
$$= \sum_{i=1}^{N} \theta_i n_i^2 + \frac{\theta_f}{\sigma_i^2} \sum_{i=1}^{N} p_i \sigma_i^2 n_i^2 - \frac{\theta_f}{\sigma_i^2} \sum_{i=1}^{N} p_i \sigma_i^2 n_f^2 + \theta_f n_f^2 - \Theta$$
$$= \sum_{i=1}^{N} \left( \frac{\theta_f p_i \sigma_i^2 n_i^2}{\sigma_i^2} + \theta_f \right) n_i^2 - \frac{\theta_f}{\sigma_i^2} D^2 - \Theta$$
Summary of Gamma P/L for Dispersion Trade

\[
\text{Gamma P/L} = \sum_{i=1}^{N} \left( \frac{\theta_i p_i \sigma_i^2 n_i^2}{\sigma_i^2} + \theta_i \right) n_i^2 - \frac{\theta_i}{\sigma_i^2} D^2 - \Theta
\]

Example: “Pure long dispersion” (zero idiosyncratic Gamma):

\[
\theta_i = -\frac{p_i \sigma_i^2}{\sigma_i^2} \quad \Theta = \left| \phi \right| \left( \sum_{i} \frac{n_i \sigma_i^2}{\sigma_i^2} - 1 \right) \geq \left| \phi \right| \left( \sum_{i} \frac{n_i \sigma_i^2}{\sigma_i^2} - 1 \right) > 0
\]

Payoff function for a trade with short index/long options (IVH), 2 stocks

Value function (B&S) for the IVH position as a function of stock prices (2 stocks)

In general: short index IVH is short-Gamma along the diagonal, long-Gamma for "transversal" moves
Gamma Risk: Negative exposure for ‘parallel’ shifts, positive ‘exposure’ to transverse shifts

\[ \sigma_1 = 30\% \]
\[ \sigma_2 = 40\% \]
\[ \rho_{12} = .5 \]

Gamma-Risk for Baskets

\[ X_i = \frac{\Delta S_i}{S_i} \]
\[ Y = \frac{\Delta l}{l} \]

\[ D = \sum_{i=1}^{N} p_i (X_i - Y)^2 \]

\[ D/Y^2 = \sum_{i=1}^{N} p_i (X_i/Y - 1)^2 \]

D= Dispersion, or cross-sectional move, 
D/(Y*Y)= Normalized Dispersion
Vega Risk

Sensitivity to volatility: move all single-stock implied volatilities by the same percentage amount

\[
\text{Vega P/L} = \sum_{j=1}^{M} \text{Vega}_j \Delta \sigma_j + \text{Vega}_i \Delta \sigma_i
\]

\[
= \sum_{j=1}^{M} (NV)_j \frac{\Delta \sigma_j}{\sigma_j} + (NV)_i \frac{\Delta \sigma_i}{\sigma_i}
\]

\[
= \left[ \sum_{j=1}^{M} (NV)_j + (NV)_i \right] \frac{\Delta \sigma}{\sigma}
\]

\[
NV = \text{normalized vega} = \sigma \frac{\partial V}{\partial \sigma}
\]

Market/Volatility Risk

- Short Gamma on a perfectly correlated move
- Monotone-increasing dependence on volatility (IVH)
``Rega’’: Sensitivity to correlation

\[ p_i \rightarrow p_i + \Delta \rho \quad i \neq j \]

\[ \sigma_i^j \rightarrow \sum_{j=1}^n \rho_{ij} \sigma_i \sigma_j + \left( \sum_{j=1}^n \rho_{ij} \sigma_j \right) \Delta \rho \]

\[ \Delta \sigma_i^j = \left( \sigma_i^{(I)} \right)^2 - \left( \sigma_i^{(II)} \right)^2 \Delta \rho, \quad \sigma_i^{(I)} = \sum_{j=1}^n \rho_{ij} \sigma_j, \quad \sigma_i^{(II)} = \sqrt{\sum_{j=1}^n \rho_{ij}^2 \sigma_j^2} \]

\[ \frac{\Delta \sigma_i^j}{\sigma_i^j} = \frac{1}{2} \left( \frac{\sigma_i^{(I)}}{\sigma_i^j} \right)^2 - \left( \frac{\sigma_i^{(II)}}{\sigma_i^j} \right)^2 \Delta \rho \]

Correlation P/L = \( \frac{1}{2} \langle NV \rangle _i \left( \frac{\sigma_i^{(I)}}{\sigma_i^j} \right)^2 - \left( \frac{\sigma_i^{(II)}}{\sigma_i^j} \right)^2 \Delta \rho \)

Rega = \( \frac{1}{2} \left( \frac{\sigma_i^{(I)}}{\sigma_i^j} \right)^2 - \left( \frac{\sigma_i^{(II)}}{\sigma_i^j} \right)^2 \times \langle NV \rangle _i \)

---

Market/Correlation Sensitivity

- Short Gamma on a perfectly correlated move
- Monotone-decreasing dependence on correlation
Valuation Method I: Weighted Monte Carlo

- Simulate scenarios (paths) for the group of stocks that comprise the index or indices under consideration
- Simulate the cash-flows of options on all the stocks and the index options
- Select weights or probabilities on the scenarios in such a way that all options/forward prices are correctly reproduced by averaging over the paths
- Use “weighted Monte Carlo” to derive fair-value of target options and compare with market values

**MC with Non-Uniform Probabilities**

Avellaneda, Buff, Friedman, Kruk, Grandchamp: IJTAF, 1999

- SDE is used to sample the path space
  \[ dX = \sum dW + B \cdot dt \]
- SDE represents Bayesian prior, e.g. subjective probability

- Reweighted probabilities reflect prices of traded securities - Arrow-Debreu probabilities
**MC with Non-Uniform Probabilities**

Avellaneda, Buff, Friedman, Kruk, Grandchamp: IJTA, 1999

- SDE is used to sample the path space
  \[ dX = \sum dW + B \cdot dt \]
- SDE represents Bayesian prior, e.g. subjective probability

- Reweighted probabilities reflect prices of traded securities - Arrow-Debreu probabilities

**Computation of weights:**

Max-Entropy Method

Determine probabilities by maximizing entropy or minimizing cross-entropy with respect to prior

Maximize \( H(p) = -\sum_{i=1}^{V} p_i \ln p_i \)

Subject to

\[
\begin{pmatrix}
C_1 \\
C_2 \\
\vdots \\
C_N
\end{pmatrix}
= \begin{pmatrix}
g_{11} & \cdots & g_{1V} \\
\cdots & \ddots & \cdots \\
g_{N1} & \cdots & g_{NV}
\end{pmatrix}
\begin{pmatrix}
p_1 \\
p_2 \\
\vdots \\
p_V
\end{pmatrix}
\]

Risk-neutral pricing probabilities

Cash-flow matrix

Market prices of single-stock options
Example of Pricing with WMC

Index Market Vols vs. Model Vols: January 03 expiration

Another Valuation Example with WMC (From Aug 2002, front month)
Another Valuation Example with WMC (From Aug 2002, second month)

![Implied vol Expiration Oct02](image)

Another Valuation Example with WMC (From Aug 2002, third month)

![Implied vol Expiration Nov02](image)
Another Valuation Example with WMC (From Aug 2002, 4th month)

Valuation Method II: (WKB)
Steepest-Descent Approximation

- Improvement on Standard Volatility Formula for Index Options
  \[ \sigma_i^2 = \sum_{j=1}^{N} p_j^2 \sigma_j^2 + \sum_{i<j} p_i p_j \sigma_i \sigma_j \rho_{ij} \quad (*) \]

- Assume that the correlation is given

- Use markets on single-stock volatilities taking into account volatility skew

- How can we integrate volatility skew information into (*)?
Steepest-Descent Approximation

- Define a risk-neutral 1-factor model for the index process
  \[ \frac{dI}{I} = \sigma_I(t) dW + \mu_I(t) dt \]

- Local index vol= conditional expectation of local variance (rigorous)
  \[ \sigma^2(I,t) = \mathbb{E}\left[ \sum_{j=1}^{N} \sigma_j(S_j(t),t) \sigma_j(S_j(t),t) \rho_{j,\mu_j} \sigma_j(S_j(t),t) \sum_{j=1}^{N} w_j S_j(t) = 1 \right] \]

- Approximate this conditional expectation using the most likely stock configuration \(\{S_1^*, ..., S_n^*\}\) given that \(\sum_{j=1}^{N} w_j S_j(t) = I\)
  \[ \sigma^2(I,t) \approx \sum_{j=1}^{N} p_j \sigma_j(S_j^*, t) \sigma_j(S_j^*, t) \]

Steepest descent vs. Market vs. WMC (Aug 20, 2002, front month)

Expiration: Sep 02

- BidVol
- AskVol
- WMC vol
- Steepest Desc
Gargoyle Dispersion Fund

- Joint venture between Gargoyle Strategic Partners and Marco Avellaneda (manager)
- Started Trading: May 2001
- Uses proprietary system to detect trades and executes electronically and through network of brokers in 5 U.S. exchanges
- 1 FT junior trader, 3 PT senior traders, 1 FT risk manager
ROI May01-Oct02

Trading History: Monthly Returns

-16.17%  -7.12%  -2.02%  -0.66%  5.20%  3.27%  3.09%  3.76%  6.09%  3.27%  0.49%  2.16%  1.90%  1.82%  2.43%  -5.85%  -2.43%  -0.07%  -10.87%  -0.67%  12.54%  3.86%  9.66%  10.10%  7.63%  5.18%  5.58%  13.97%  9.18%  7.40%  6.13%  6.06%  3.43%  7.24%  13.97%  12.40%  15.73%  20.00%  10.00%  5.00%  0.00%  -5.00%  -10.00%  -15.00%  -20.00%  S&P 500

Gargoyle Dispersion Fund

MAY-01  JUN-01  JUL-01  AUG-01  SEP-01  OCT-01  NOV-01  DEC-01  JAN-02  FEB-02  MAR-02  APR-02  MAY-02  JUN-02  JUL-02  AUG-02  SEP-02  OCT-02
Dispersion Fund Performance

Trading Period: 15 months

Cumulative ROI* since inception: 28.33%

Annualized Rate of Return: 22.65%

Annualized Standard Deviation: 26.59%

Worst monthly loss: August 02, -16%

Correlation with S&P 500: 35%

Correlation with VIX Index: -33%

* After paying brokerage fees and commissions, etc
DJX Correlation Blowout, July 2002
## Conclusions

- Dispersion trading: a form of "statistical correlation arbitrage"
- Sell correlation by selling index options and buying options on the components
- Buy correlation by buying index options and selling options on the components
- "Convergence trading" style.
- Price discovery using model and market data on vol skews
- Sophisticated trading strategy. Potentially very profitable, with moderate (but not low) risk profile.