1. (a) How do we compute a new iterate \( x_{k+1} \) from \( x_k \) if we want solve \( f(x) = 0 \) by Newton’s method?

(b) Consider \( f(x) = x \sin(\pi x) \). This function has zeros at 0, +1, −1, +2, −2, etc. What can we say about the rate of convergence of finding these zeros of \( f(x) = 0 \) if we start our iterations close to one of the roots. Will any of them take more iterations than others? In particular, have a look at the root at 0 and at 1.

\[
x_{k+1} = x_k - \frac{f(x_k)}{df(x_k)/dx}.
\]

Newton’s method will converge quadratically to a root \( \xi \) provided that \( df(\xi)/dx \neq 0 \). Therefore we will have rapid convergence to all the roots of the given function which differ from 0. The function has a double root at 0 and Newton’s method will converge much slower to that root. Compare Problem 1.6 in the textbook.
2. Linear systems of algebraic equations, $Ax = b$, where $A$ is a square, invertible matrix of order $n$ and $x$ and $b$ are $n$–vectors are solved by using partial pivoting. Recall that then rows of the matrix are exchanged to assure that the elements of the lower triangular matrix $L$ all satisfy $|\ell_{ij}| \leq 1$.

Consider the matrix

\[
\begin{pmatrix}
1 & 2 & 3 \\
4 & 5 & 6 \\
7 & 8 & 9 \\
\end{pmatrix}
\]

If we apply Gaussian elimination with partial pivoting of this matrix, in what order will the rows of the matrix emerge? In other words, what is the permutation matrix $P$. Just do the minimal computations to answer this specific question.

In the first step, we will exchange the first and the third rows. We then have to compute and compare the new values of the elements in position 22 and 23. We then find that we need to exchange the second and third rows. Therefore the original row 3 ends up on top, followed by the original row 1 and then the original row 2.