Assignment Set 2, UA-MATH0252-1

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February 17, 2014

The following assignments are due on February 28 at 12:00 noon. No homework will be accepted after that time. You should give me your homework in class or put it under my office door (WWH612). Do not use my mailbox in the WWH lobby.

You should turn in your MATLAB/OCTAVE programs and print-outs which shows how they work for different input data.

Turn in a listing of your MATLAB programs as well as output from selected runs. Always use format long e in MATLAB. Note that in MATLAB, all numbers are represented with approximately 16 decimals; we will discuss computer arithmetic in the middle of the term.

1. Problem 2.1 from the text book.
2. Problem 2.4 from the text book.
4. Problem 2.15 from the text book.
5. A banded, square matrix $A$, with band width $k$ satisfies $a_{ij} = 0$ for all $i$ and $j$ such that $|i - j| > k$.

Suppose we wish to develop a special Gaussian elimination program to solve $Ax = b$ for banded matrices. How much storage is required if pivoting is not required? How much is needed if we allow pivoting?

6. Write a MATLAB program for Gaussian elimination of banded matrices, which uses only one loop; do so by exploiting the possibilities of working with matrices rather than with individual matrix elements.

Make sure that you do not unnecessarily store elements that will remain zero throughout.

If possible, also develop a version which allows for partial pivoting.

Show, by numerical experiments, that your program works correctly by generating matrices (and right hand sides) some of which require partial
pivoting, and comparing your solutions with those obtained by MATLAB and the simplest possible script.

7. Formulate and solve the least squares problem used by Gauss in his 1850 lectures. Use MATLAB and the simplest possible script. Compute the least squares solution as well as the maximum error of the solution.

The data is as follows:

- \( Q = P + 64.334 \)
- \( R = P + 349.366 \)
- \( R = Q + 283.596 \)
- \( S = Q + 206.580 \)
- \( S = R - 76.108 \)
- \( T = R + 648.427 \)
- \( T = S + 719.612 \)

\( S, T, Q, R, \) and \( P \) represent the elevation of five towns in Northern Germany. The solution represents the difference in elevation of the towns. Note that you cannot compute the actual elevation with respect to sea level from the data given.

8. A square matrix \( A \) is strictly column diagonally dominant if

\[
\sum_{i \neq j} |a_{ij}| < |a_{jj}|, \quad \text{for } i = 1, \ldots, n.
\]

Prove that pivoting for Gaussian elimination is not needed for a linear system with such a matrix.

Hints: The tridiagonal case is discussed in the textbook. First prove that no pivoting is need in the first step. Then prove that the matrix that remains to be \( LU \)-factored is also strictly diagonally dominant.

9. Assume that a square matrix is strictly row diagonally dominant:

\[
\sum_{j \neq i} |a_{ij}| < |a_{ii}|, \quad \text{for } i = 1, \ldots, n.
\]

Show that any such matrix is invertible.