1. (a) Under what condition will Newton’s method converge slowly even if we start from with a quite accurate initial approximation \( x_0 \) of a root of \( f(x) = 0 \)?

The convergence is slow if \( f' \) vanishes at the root. See problem 1.6 in the textbook.

(b) Are there situations for which the bisection algorithm to find a root of \( f(x) = 0 \) will not work?

We cannot get this method to work if there is no change of sign, e.g., if we have a double root.

2. (a) In about ten words: What is Gaussian elimination? In particular, for what problems is it used?

Gaussian elimination is a method for solving linear systems of algebraic equations of the form \( Ax = b \), where \( A \) is a square matrix. The linear system is turned into upper triangular form by a series of row operations.

(b) How much work would that require if the matrix is \( n \times n \)?

The work – the number of arithmetic operations – grows cubically in \( n \).

(c) Gaussian elimination can produce quite inaccurate results. Explain what properties of the matrix can cause this to happen.

We can get inaccurate results if the matrix is ill-conditioned, i.e., \( \|A\|\|A^{-1}\| \) is large. See the end of section 2.7 in the textbook.