Homework set 9: Due Friday November 14 at 12 noon.

Homework should be given to me (or my possible replacement) in class or put under my office door. **Do not put it in my lobby mail box.** No credit will be given for homework turned in late.

1. Compute $I_2$ and $I_3$ where
   $$I_m := \int_{-\infty}^{\infty} \frac{dx}{1 + x + x^2 + \ldots + x^{2m}}.$$

2. Show that the inverse Laplace transform of
   $$\frac{2s - 2}{(s + 1)(s^2 + 2s + 5)}$$
   equals $e^{-t}(\cos(2t) + \sin(2t) - 1)$.

3. Show that
   $$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} \text{ and } \sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}.$$

4. Let $f(z)$ be analytic inside and on a positively oriented simple closed contour $C$. $f(z)$ differs from zero on $C$ and has $n$ zeros $z_k$ of multiplicity $m_k$ inside $C$. Show that
   $$\int_C \frac{zf'(z)dz}{f(z)} = 2\pi i \sum_{k=1}^{n} m_k z_k.$$

5. Show that the number of roots, counting multiplicity, of
   $$2z^5 - 6z^2 + z + 1 = 0$$
   in the annulus $1 \leq |z| < 2$ equals 3.