Homework set 4: Due Friday October 10 at 12 noon.

Homework should be given to me (or my possible replacement) in class or put under my office door. Do not put it in my lobby mail box. No credit will be given for homework turned in late.

1. Show that $|exp(-2z)| < 1$ if and only if Re$z > 0$.

2. Let Log$z$ be the principal value of the function log$z$.
   Show that Log$(i^3) \neq 3 \log(i)$.
   Does a statement Log$(i^3) = 3 \log(i)$ make any sense if interpreted correctly?

3. Find all $z$ such that cos$z = \sqrt{2}$.

4. Show that $\sin^{-1}(z) = \frac{1}{i} \log(\sqrt{1-z^2} + iz)$.
   Explain why this is a multi-valued function.
   Show that this function can be made single-valued by two branch cuts along the real axis from $-\infty$ to $-1$ and from $1$ to $+\infty$.

5. Let $\sum_{n=0}^{\infty} a_n z^n$ and $\sum_{n=0}^{\infty} b_n z^n$ be two power series with a positive radius of convergence $R$. Let $\sum_{n=0}^{\infty} c_n z^n$ be a power series with $c_n := \sum_{k=0}^{n} a_k b_{n-k}$. Show that this third power series have a radius of convergence of at least $R$.

6. Consider the function
   $$f(z) := \frac{z}{e^z - 1}.$$ 
   Show that it is analytic in a neighborhood of the origin.
   Find the first four terms in a power series expansion of $f(z)$. 

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