1. Consider an analytic function \( f(z) \) in a domain in the complex plane, which has a constant modulus \( |f(z)| \). Show that \( f(z) \) must then be a constant function.

2. Express \( \cos 3\phi, \cos 4\phi, \) and \( \cos 5\phi \) in terms of \( \cos \phi \) and \( \sin \phi \).

3. Consider the root of \( z^n = 1 \) given by \( \omega = \cos \frac{2\pi}{n} + i\sin \frac{2\pi}{n} \). For which integers \( m \) is
   \[
   1 + \omega^m + \omega^{2m} + \cdots + \omega^{(n-1)m} = 0?
   \]

4. Consider a convergent sequence \( \{z_n\} \) of complex numbers such that \( |z_n - z| \to 0 \). Show that the sequence \( \{Z_n\} \) of the corresponding points on the unit sphere of the stereographic projection also converges.
   Is it also true that the convergence of \( \{Z_n\} \to Z \) implies the convergence of the corresponding sequence in the complex plane?
   If possible consider all cases including \( z = \infty, Z = (0, 0, 1) \).

5. Show that, except for at \( z = 0 \), \( \log |z| \) is a harmonic function and use calculus to find its conjugate harmonic function.